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Rough, Jimmie Lynn

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RANDOM NEUTRON FLUX VARIATIONS
IN A SUBCRITICAL ASSEMBLY

JIMMIE LYNN ROUGH

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RANDOM NEUTRON FLUX VARIATIONS
IN A SUBCRITICAL ASSEMBLY

by

Jimmie Lynn Rough
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A Thesis Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
MASTER OF SCIENCE

Major Subject: Nuclear Engineering

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INTRODUCTION

The subcritical assembly has been used as an educational device to demonstrate the principles of nuclear science and engineering, and for research in experimental reactor engineering. There are several advantages in the use of a subcritical assembly. The assembly is small in size and relatively inexpensive in materials, construction, and required apparatus. Only nominal shielding is required due to the low activity involved. Lattice parameters can be varied more easily than in a critical assembly.

The kinetic equations of the assembly are time dependent, non-linear differential equations which may be linearized in the derivation of the transfer function. The method of experimental evaluation of the transfer function is to vary one or more of the parameters in such a way that the limits within which the linearization is valid are not exceeded. This variation of the parameters is called "forcing". The usual methods of forcing in a subcritical assembly are varying the strength of the neutron source or the reactivity in a periodic manner. The minimum periodic input signal to which the response can be measured is determined by the amplitude of the random or stochastic variations in the steady state flux level due to the inherent statistical nature of the fission process. It is possible, in a system with large stochastic variations in the neutron flux, to exceed the range of line-

arity of the kinetic equations by making the magnitude of the input signal large enough to obtain a readable response. A technique has been developed, from studies of stochastic processes such as Brownian Motion, which makes it possible to evaluate the transfer function of an assembly. The stochastic or random variation in the flux is considered to be the response of the system to stochastic source forcing due to the statistical nature of the basic phenomena involved. This method of evaluation of the transfer function should be least likely to invalidate the linear approximations made.

REVIEW OF THE LITERATURE

A great amount of effort has been expended in the derivation and evaluation of critical assembly transfer functions. One of the primary reasons was the need for an automatic control system to maintain a given power level in a reactor. The similarity of the transfer function of a reactor to that of a servomechanism control system had been noted, and it was determined that this type of system was suitable for reactor control.

Owens, Crever, and Pigott (13) in 1949 proposed the concept of treating a critical assembly as a "black box" to which a feedback control system could be applied in order to maintain a desired power level. The proposal was based on the observation that the roots of the kinetic equation of the assembly were negative. This indicated that the critical assembly transfer function was the same as the transfer function for a minimum phase shift network, and that the transfer characteristic could be represented by a Bode diagram. In that same year Franz (5) presented a more detailed derivation of the transfer function and showed that the standard techniques of servomechanism theory applied. Harrer, Boyar, and Krucoff (9) evaluated the transfer function of the CP-2 reactor by sinusoidally varying the reactivity, and showed that the transfer function could be used in the design of servo loops or regulating systems which include the reactor.

the same time, the government has the authority to make laws

which will not interfere with trade. We do not desire a situation where different countries have to make their own

regulations for our trade and similarly we do not want regulations by one country to affect another's economy. So it is important that there is a system of rules and regulations which are agreed upon by all countries and are not arbitrary or discriminatory. This would help to ensure that there is no abuse of power and that the principles of free trade are respected.

It is also important that the rules are clear and simple so that they can be easily understood by all countries involved. This would help to prevent disputes between countries over what constitutes a violation of the rules.

It is also important that the rules are fair and do not discriminate against certain countries or industries. This would help to ensure that all countries have equal opportunities to compete in the world market.

It is also important that the rules are transparent and do not favor certain countries or industries. This would help to prevent corruption and ensure that the rules are applied fairly and consistently across all countries.

It is also important that the rules are flexible and can be changed as needed. This would help to ensure that the rules remain relevant and effective over time. It is also important that the rules are easy to understand and implement. This would help to prevent confusion and ensure that the rules are followed by all countries involved.

Mockup assemblies have been utilized for the determination of the transfer function of proposed reactors. Boland, Smith, and Rice (3) measured the zero power transfer function of a fast critical assembly, ZPR-III mockup assembly of EBR-I, Mark III, by determining the change in power level due to oscillation of the reactivity.

Refinement in the techniques of testing and the evaluation of transfer functions are constantly sought, and the subcritical assembly provides a relatively simple vehicle for such investigations. A subcritical assembly was utilized by Axtmann, Dessauer, and Parkinson (1) for reactivity measurements and testing of pile components. The transfer function of the Iowa State University subcritical assembly was initially evaluated by Ricci (14) utilizing the method of source forcing.

A new technique for evaluation of the transfer function of a nuclear reactor operating at a steady state by an analysis of the random, or stochastic, nature of the neutron population has been developed recently. Moore (11) has shown mathematically that the square of the modulus of the transfer function of a reactor is proportional to the Fourier transform of the autocorrelation function which is equal to the power spectral density function of the system. This was verified experimentally by Cohn (4) and by Griffin and Lundholm (8). Velez (16) evaluated the transfer function of the Ford Nuclear

Reactor and the subcritical assembly at the University of Michigan by the direct measurement of the autocorrelation function, but the results were reported to be inconclusive in the case of the subcritical assembly. All references found describing the evaluation of the transfer function by measurement of the power spectral density function pertained to critical assemblies.

DEVELOPMENT OF THE KINETIC EQUATIONS

The kinetic equations of the subcritical assembly can be derived from the general diffusion equation (6, p. 223 and 7, p. 101)

$$D \nabla^2 \phi - \sum_a \phi + S = \frac{\partial n}{\partial t} = \frac{1}{v} \frac{\partial \phi}{\partial t} \quad (1)$$

which is applicable to that region of a subcritical assembly away from the extraneous neutron source where most of the neutrons are fission neutrons. The symbols used are defined in the table of symbols. The first term represents the loss of thermal neutrons due to leakage from the assembly; the second term is the loss of thermal neutrons due to absorption; the third term is the neutron production or source term; and the right side of the equation is the non-equilibrium change in neutron population with time.

The source term is made up of prompt and delayed fission neutrons, and neutrons from an external source. It has been shown (7, p. 193) that the number of fast fission neutrons produced is $\frac{k_\infty}{p} \sum_a \phi$, and further (6, p. 226) that if β is the total fraction of delayed neutrons the prompt neutron source term S_p is

$$S_p = (1 - \beta) k_\infty \sum_a \phi e^{-B^2 T} \quad (2)$$

There are six known groups of delayed neutrons that account for 0.64 per cent of the thermal neutron production.

These delayed neutrons accompany the radioactive decay of certain fission products, called neutron precursors, which exist in an excited state. If β_i is the fraction of each delayed neutron group then the formation of the precursors of the group would be $\beta_i \frac{k_\infty}{p} \sum_a \phi$. When the concentration of a precursor group is denoted by c_i and the decay constant as λ_i , the rate of decay would be $\lambda_i c_i$. The net rate of formation of precursors to the i -th delayed neutron group would then be

$$\frac{\partial c_i}{\partial t} = \beta_i \frac{k_\infty}{p} \sum_a \phi - \lambda_i c_i \quad (3)$$

The total rate of production of delayed neutrons is $\sum_{i=1}^6 \lambda_i c_i$, and when multiplied by the resonance escape probability, p , and the slowing down non leakage probability, $e^{-B^2 \tau}$, the result is the delayed thermal neutron source term

$$S_d = p e^{-B^2 \tau} \sum_{i=1}^6 \lambda_i c_i \quad (4)$$

Table 1. Table of symbols

B^2	Buckling	cm.^{-2}
β	Fraction of delayed neutrons	
β_i	Fraction of i -th group of delayed neutrons	
C	Number of delayed neutron precursors in the assembly	
c_i	Number of the i -th group of delayed neutron precursors in the assembly	

and the species' additional nest resources. However, they will need to be taken into account. Without them, nest-hunting would be difficult since the majority will be $\frac{1}{2}$ miles below the nest. The problem may be minimized with small areas searched sequentially to accommodate nest sites. $\sum_{k=1}^n \frac{1}{k}$ miles along miles with $\frac{1}{k}$ m. distances spaced with time \propto $\frac{1}{k}$ miles. All would have to be collected by day two and $\sum_{k=1}^n \frac{1}{k}$ m. miles total. To avoid large areas between locations $\sum_{k=1}^n \frac{1}{k}$ m. of distance

$$(1) \quad \sum_{k=1}^n \frac{1}{k} = \ln n + \frac{1}{2} + \frac{1}{12} n + \dots$$

of course leads to iteration. To keep labor with all probability agencies involved and to facilitate with time and $\sum_{k=1}^n \frac{1}{k}$ m. probability against one such location will take until about portion. Locations located will all be used.

$$(2) \quad P_{\text{find}} \propto \sum_{k=1}^n \frac{1}{k} \cdot \alpha = k^2$$

Tables in this abstract

	P_{find}
Probability of finding one bird	$\frac{1}{2}$
Probability of finding two birds	$\frac{1}{4}$
Probability of finding three birds	$\frac{1}{8}$
Probability of finding four birds	$\frac{1}{16}$
Probability of finding five birds	$\frac{1}{32}$
Probability of finding six birds	$\frac{1}{64}$
Probability of finding seven birds	$\frac{1}{128}$
Probability of finding eight birds	$\frac{1}{256}$
Probability of finding nine birds	$\frac{1}{512}$
Probability of finding ten birds	$\frac{1}{1024}$
Probability of finding eleven birds	$\frac{1}{2048}$
Probability of finding twelve birds	$\frac{1}{4096}$
Probability of finding thirteen birds	$\frac{1}{8192}$
Probability of finding fourteen birds	$\frac{1}{16384}$
Probability of finding fifteen birds	$\frac{1}{32768}$
Probability of finding sixteen birds	$\frac{1}{65536}$
Probability of finding seventeen birds	$\frac{1}{131072}$
Probability of finding eighteen birds	$\frac{1}{262144}$
Probability of finding nineteen birds	$\frac{1}{524288}$
Probability of finding twenty birds	$\frac{1}{1048576}$
Probability of finding twenty-one birds	$\frac{1}{2097152}$
Probability of finding twenty-two birds	$\frac{1}{4194304}$
Probability of finding twenty-three birds	$\frac{1}{8388608}$
Probability of finding twenty-four birds	$\frac{1}{16777216}$
Probability of finding twenty-five birds	$\frac{1}{33554432}$
Probability of finding twenty-six birds	$\frac{1}{67108864}$
Probability of finding twenty-seven birds	$\frac{1}{134217728}$
Probability of finding twenty-eight birds	$\frac{1}{268435456}$
Probability of finding twenty-nine birds	$\frac{1}{536870912}$
Probability of finding thirty birds	$\frac{1}{1073741824}$
Probability of finding thirty-one birds	$\frac{1}{2147483648}$
Probability of finding thirty-two birds	$\frac{1}{4294967296}$
Probability of finding thirty-three birds	$\frac{1}{8589934592}$
Probability of finding thirty-four birds	$\frac{1}{17179869184}$
Probability of finding thirty-five birds	$\frac{1}{34359738368}$
Probability of finding thirty-six birds	$\frac{1}{68719476736}$
Probability of finding thirty-seven birds	$\frac{1}{137438953472}$
Probability of finding thirty-eight birds	$\frac{1}{274877906944}$
Probability of finding thirty-nine birds	$\frac{1}{549755813888}$
Probability of finding forty birds	$\frac{1}{1099511627776}$
Probability of finding forty-one birds	$\frac{1}{2199023255552}$
Probability of finding forty-two birds	$\frac{1}{4398046511104}$
Probability of finding forty-three birds	$\frac{1}{8796093022208}$
Probability of finding forty-four birds	$\frac{1}{17592186044416}$
Probability of finding forty-five birds	$\frac{1}{35184372088832}$
Probability of finding forty-six birds	$\frac{1}{70368744177664}$
Probability of finding forty-seven birds	$\frac{1}{140737488355328}$
Probability of finding forty-eight birds	$\frac{1}{281474976710656}$
Probability of finding forty-nine birds	$\frac{1}{562949953421312}$
Probability of finding fifty birds	$\frac{1}{1125899906842624}$

Table 1. (Continued)

c	Concentration of delayed neutron precursors	
c_i	Concentration of i-th group of delayed neutron precursors	
D	Thermal neutron diffusion coefficient	cm.
f	Frequency	<u>cycles</u> sec.
k	Effective multiplication factor	
k_∞	Multiplication factor for an infinite assembly	
L	Neutron diffusion length in the moderator	cm.
ℓ^*	Prompt neutron lifetime in a finite assembly	sec.
ℓ_0	Prompt neutron lifetime in an infinite assembly	sec.
$\bar{\ell}$	Average neutron lifetime in a finite assembly	sec.
$\bar{\lambda}$	Average decay constant of delayed neutron precursors	sec. ⁻¹
λ_i	Decay constant of the i-th group of delayed neutron precursors	sec. ⁻¹
N	Total number of thermal neutrons in a finite assembly	
n	Number of thermal neutrons per unit volume	<u>neutrons</u> cm. ³
p	Resonance escape probability	
ϕ	Thermal neutron flux	<u>neutrons</u> cm. ² .sec.
S	Rate of production of thermal neutrons per unit volume from all source	<u>neutrons</u> cm. ³ .sec.
S_e	Total external thermal neutron source contribution to the assembly	<u>neutrons</u> sec.

Table 1. (Continued)

\mathcal{A}	External thermal neutron source contribution per unit volume	<u>neutrons</u> cm. ⁻³ sec.
\sum_a	Macroscopic absorption cross section	cm. ⁻¹
t	Time	sec.
τ	Fermi age	cm. ²
v	Thermal neutron mean velocity	cm./sec.
ω	Frequency	<u>radians</u> sec.

The contribution of the external thermal neutron source term will be assumed to be

$$S = \mathcal{A} \quad (5)$$

Substitution of Equations 2, 4, and 5 into Equation 1 yields

$$\begin{aligned} \frac{\partial n}{\partial t} &= D \nabla^2 \phi - \sum_a \phi + (1 - \beta) k_a \sum_a \phi e^{-B^2 \tau} \\ &\quad + p e^{-B^2 \tau} \sum_{i=1}^6 \lambda_i c_i + \mathcal{A} \end{aligned} \quad (6)$$

ϕ and c_i are functions of space and time, and it can be shown (6, p. 227) that the space and time variables are separable allowing the partial differential equation to be reduced to an ordinary differential equation. Consideration of the fundamental space mode only (6, p. 227 and 9, p. 33) yields

	Practical approach
standard deviation	accuracy versus ordinary standard deviation
coefficient of variation	accuracy versus relative precision
precision	accuracy versus relative precision
precision coefficient	accuracy versus relative precision
precision coefficient of variation	accuracy versus relative precision
precision coefficient of variation coefficient	accuracy versus relative precision

and some authors (e.g. Lovell and 1972) distinguish between the two methods of expressing precision.

(7)

accuracy of sampling and testing and precision of analytical methods

$$\text{accuracy} \geq 0.95 \times 1 + k_1 \geq 1.95 - \frac{0.5}{1}$$

for

$$\frac{\text{precision}}{\text{precision}} \leq \frac{1}{k_1} \quad \frac{\text{precision}}{\text{precision}} \leq 1$$

whereas the first term has enough precision that the 5% difference will probably not exceed 0.05 (95% to 90%) of the value of the analytical test statistic and coincide with the confidence limit settings recommended previously by about 100 to 150 ppm (0.01 to 0.02% relative precision).

the equality

$$\nabla^2 \phi = -B^2 \phi \quad (7)$$

The diffusion length is defined by

$$L^2 = \frac{D}{\sum_a} \quad (8)$$

Rearrangement of the equation yields

$$D = L^2 \sum_a \quad (9)$$

The mean life of thermal neutrons in an infinite medium is defined by

$$\ell_o \equiv \frac{\lambda a}{v} = \frac{1}{\sum_a v} \quad (10)$$

When rearranged this relation becomes

$$\sum_a = \frac{1}{\ell_o v} \quad (11)$$

The flux, the effective multiplication factor in a finite medium, and the prompt neutron lifetime in a finite medium are defined by

$$\phi = nv \quad (12)$$

$$k = \frac{k_\infty e^{-B^2 \tau}}{1 + L^2 B^2} \quad (13)$$

$$\ell^* = \frac{\ell_o}{1 + L^2 B^2} \quad (14)$$

The combination of Equations 13 and 14 yields

the first two terms in the expansion of \hat{H}_0 are

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) \quad (1)$$

where $V_0(r)$ is the potential energy of the system.

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) \quad (1)$$

The third term in the expansion of \hat{H}_0 is the spin-orbit coupling term.

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} \quad (2)$$

The fourth term in the expansion of \hat{H}_0 is the spin-orbit coupling term.

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

The fifth term in the expansion of \hat{H}_0 is the spin-orbit coupling term.

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

The sixth term in the expansion of \hat{H}_0 is the spin-orbit coupling term.

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

The seventh term in the expansion of \hat{H}_0 is the spin-orbit coupling term.

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

$$\hat{H}_0 = \frac{\hbar^2}{2m} \nabla^2 + V_0(r) + \lambda \vec{S} \cdot \vec{r} + \frac{e}{m} \vec{A} \cdot \vec{p} \quad (3)$$

$$k_{\infty} = \frac{\ell_0 k}{\ell^* e^{-B^2 \tau}} \quad (15)$$

The substitution of Equations 7, 9, 11, 12, and 15 into Equation 6 yields

$$\begin{aligned} \frac{dn}{dt} = & - (L^2 B^2 + 1) \frac{n}{\ell_0} + (1 - \rho) k \frac{n}{\ell^*} \\ & + p e^{-B^2 \tau} \sum_{i=1}^6 \lambda_i c_i + s \end{aligned} \quad (16)$$

The combination of Equation 14 with Equation 16 and rearranging yields

$$\frac{dn}{dt} = [k(1 - \rho) - 1] \frac{n}{\ell^*} + p e^{-B^2 \tau} \sum_{i=1}^6 \lambda_i c_i + s \quad (17)$$

When Equations 11, 12, and 15 are substituted into Equation 3 the result is

$$\frac{dc_i}{dt} = \rho_i \frac{k n}{p e^{-B^2 \tau} \ell^*} - \lambda_i c_i \quad (18)$$

Equations 17 and 18 are the coupled kinetic equations of the subcritical assembly.

It should be noted at this point that in arriving at Equation 17 the assumption has been made that the external source contributes a different number of thermal monoenergetic neutrons to each unit volume of the subcritical assembly, the number decreasing with height in the assembly. The kinetic equations can also be derived by considering the total neutron population rather than the neutron population

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 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = 0$
 where x_i is the i th coordinate and L is the Lagrangian with
 \dot{x}_i being a velocity
 $\ddot{x}_i = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_i} \right)$
 we can write the equations of motion as

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = 0$$

 which are called the Euler-Lagrange equations of motion
 and x_i is called a generalized coordinate
 (10.1)

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = 0$$

 with the initial conditions fixed by choosing initial values for the coordinates
 and velocities at time $t=0$ and allowing the time t to increase.
 Let's consider some specific cases and the mechanics will tell us what
 the equations of motion are. We start with the case of a particle moving
 in one dimension. Let's say it's moving along the x -axis. The
 potential energy is zero and the Lagrangian will be
 $L = m \dot{x}^2 / 2$. The equations of motion are

$$m \ddot{x} = 0$$

 or $\ddot{x} = 0$. This means that the position of the particle will
 not change over time. In other words, the particle is stationary.

on a per unit volume basis.

Consider that at a particular time the assembly contains N neutrons, and the effective reproduction constant, k , is less than one. The prompt neutron lifetime is λ^* , the prompt neutron reproduction constant is k_p , and the rate of decrease of neutron population in the assembly is $(k_p - 1) N/\lambda^*$, if neutrons are not supplied to the assembly from an external source. Absorption and leakage account for the decrease. If β is the fraction of delayed neutrons then the prompt neutron reproduction constant, k_p , can be replaced by $k(1 - \beta)$. The rate of decrease of neutron population with time is then given by

$$\frac{dN}{dt} = [k(1 - \beta) - 1] N/\lambda^* \quad (19)$$

Neutrons are supplied to the assembly by an external source and six groups of delayed neutrons. The total number of delayed neutrons of the i -th group stored in the fission products of the assembly at any time is C_i . If the decay constant of the nuclei containing the i -th group of delayed neutrons is λ_i , the neutrons of that group are emitted at a rate $\lambda_i C_i$ as fast neutrons. These fast delayed neutrons must be corrected for fast non-leakage, $e^{-B^2 \tau}$, and resonance escape probability, p . Summation of the terms in the non-equilibrium rate equation for the assembly yields

$$\frac{dN}{dt} = [k(1 - \rho) - 1] \frac{N}{\lambda^*} + pe^{-B^2\tau} \sum_{i=1}^6 \lambda_i c_i + S_e \quad (20)$$

The total number of thermal neutrons being produced per unit time is $\frac{k N}{\lambda^*}$ of which a fraction ρ are delayed. The delayed neutrons are not lost by leakage or absorption until after they are emitted from the fission products, and hence the number of delayed neutrons stored in the fission products is greater than $\frac{k N}{\lambda^*}$ by the factor $\frac{1}{pe^{-B^2\tau}}$. The non-equilibrium equation for the i -th group of delayed neutrons is

$$\frac{d c_i}{dt} = \frac{\rho k N}{p e^{-B^2\tau}} - \lambda_i c_i \quad (21)$$

Equations 20 and 21 are the same as Equations 17 and 18 except that they are based upon the total number of neutrons in the assembly, and the external source term then accounts for all of the neutrons supplied to the assembly. The neutron flux at any point in the assembly can be considered to be proportional to the total number of neutrons in the assembly at a particular instant. The use of either set of coupled kinetic equations in the derivation of the transfer function will yield the same result.

$$(10) \quad \int_0^T \left| \frac{d}{dt} \int_{\Omega} u(t,x) \cdot \nabla \left(\tau_{\lambda}(u(t,x)) \right) \right|^2 dx dt$$

the previous subsections about the solution $u(t,x)$ we can get the desired estimate by using the energy inequality and the local compactness lemma. Since the solution $u(t,x)$ has been bounded in $L^{\infty}(\Omega \times [0,T]; H^1(\Omega))$, we can choose a subsequence if necessary such that $u(t,x)$ converges weakly in $L^2(\Omega \times [0,T]; H^1(\Omega))$ and strongly in $L^2(\Omega \times [0,T]; L^2(\Omega))$. Then we can get the following estimate.

At last from formula (9) we can get estimates (10).

$$(10) \quad \int_0^T \left| \frac{d}{dt} \int_{\Omega} u(t,x) \cdot \nabla \left(\tau_{\lambda}(u(t,x)) \right) \right|^2 dx dt$$

and (11) and (12) are obtained by using (10) and the local compactness lemma. So we can find some local compactness lemma which states that every bounded sequence in $L^2(\Omega \times [0,T]; H^1(\Omega))$ has a subsequence which converges weakly in $L^2(\Omega \times [0,T]; L^2(\Omega))$. Then we can get the following estimate. Since the solution $u(t,x)$ has been bounded in $L^{\infty}(\Omega \times [0,T]; H^1(\Omega))$, we can choose a subsequence if necessary such that $u(t,x)$ converges weakly in $L^2(\Omega \times [0,T]; H^1(\Omega))$ and strongly in $L^2(\Omega \times [0,T]; L^2(\Omega))$. Then we can get the following estimate.

$$(11) \quad \int_0^T \left| \frac{d}{dt} \int_{\Omega} u(t,x) \cdot \nabla \left(\tau_{\lambda}(u(t,x)) \right) \right|^2 dx dt$$

DERIVATION OF THE TRANSFER FUNCTIONS

The kinetic equations and the resultant transfer function can be varied by varying the source or reactivity in a predetermined manner. This variation is usually sinusoidal when purposely introduced. However, the source strength and reactivity may vary even at "steady state" conditions. This is due to the fluctuation in the number of neutrons available to the assembly caused by the natural statistical fluctuations in the rates of neutron absorption and fission. The resultant random variations in the neutron flux make possible the evaluation of the transfer function by proper analysis at steady state.

Transfer Function with Source Variation

For steady state conditions (using zero subscripts)

Equations 17 and 18 become

$$\frac{dn}{dt} = 0 = [k(1 - \rho) - 1] \frac{n_0}{\lambda^*} + p e^{-B^2 T} \sum_{i=1}^6 \lambda_i c_{i0} + s_0 \quad (22)$$

$$\frac{dc_i}{dt} = 0 = \frac{\rho k}{p e^{-B^2 T}} \cdot \frac{n_0}{\lambda^*} - \lambda_i c_{i0} \quad (23)$$

Consider a small variation δs to be superimposed on s_0
so that

$$\begin{aligned} s &= s_0 + \delta s \\ n &= n_0 + \delta n \\ c_i &= c_{i0} + \delta c_i \end{aligned} \quad (24)$$

theoretical research and its application.

positive outcomes should vary with one's attitudes towards self-

-image or self-worth. Specifically, the outcome will depend upon the individual's own positive or negative self-esteem. As a matter of fact, individuals with high self-esteem are more likely to succeed than those with low self-esteem. This is because the individual with high self-esteem is more likely to believe in his/her abilities and to have more confidence in his/her own worth. In contrast, individuals with low self-esteem are less likely to believe in their own worth and to have less confidence in their abilities. Therefore, the outcome will depend upon the individual's own positive or negative self-image and the individual's own self-esteem.

STATE QUOTE

positive self-image is the outcome of success.

Individuals with high self-esteem are more successful than those with low self-esteem.

(iii) $\frac{d}{dt} \int_{0}^{t_0} g(t-s) f(s) ds = \int_{0}^{t_0} [g(t-s)f(s)]' ds = g(t-t_0)f(t_0)$

(iv) $\int_{0}^{\infty} g(t-s) f(s) ds = \int_{0}^{\infty} g(t-s) ds \int_{0}^{\infty} f(s) ds$

For an explanation of each equation, refer to the text book.

Ex. 1. $\int_{0}^{\infty} e^{-st} \sin t dt$

(v) $\int_{0}^{\infty} e^{-st} \cos t dt$

Ex. 2. $\int_{0}^{\infty} e^{-st} dt$

Substitution of Equation 24 into Equations 17 and 18 yields,

$$\frac{dn}{dt} = \frac{d\delta n}{dt} = [k(1 - \beta) - 1] \frac{(n_o + \delta n)}{\ell^*} + p e^{-B^2 \tau} \sum_{i=1}^6 \lambda_i (c_{i0} + \delta c_i) + \delta c_o + \delta \alpha \quad (25)$$

and

$$\frac{dc_i}{dt} = \frac{d\delta c_i}{dt} = \frac{\beta_i k}{p e^{-B^2 \tau}} \frac{(n_o + \delta n)}{\ell^*} - \lambda_i (c_{i0} + \delta c_i) \quad (26)$$

When the terms of Equations 25 and 26 are expanded, the steady state components of Equations 22 and 23 are collected and set equal to zero, the resultant equations are

$$\frac{d\delta n}{dt} = \frac{k-1}{\ell^*} \delta n - \frac{k\beta}{\ell^*} \delta n + p e^{-B^2 \tau} \sum_{i=1}^6 \lambda_i \delta c_i + \delta \alpha \quad (27)$$

$$\frac{d\delta c_i}{dt} = \frac{\beta_i k}{p \ell^* e^{-B^2 \tau}} \delta n - \lambda_i c_i \quad (28)$$

by transforming to Laplace notation Equations 27 and 28 become

$$s \delta n(s) = \frac{k-1}{\ell^*} \delta n(s) - \frac{k\beta}{\ell^*} \delta n(s) + p e^{-B^2 \tau} \sum_{i=1}^6 \lambda_i \delta c_i(s) + \delta \alpha(s) \quad (29)$$

$$s \delta c_i(s) = \frac{\beta_i k}{p \ell^* e^{-B^2 \tau}} \delta n(s) - \lambda_i \delta c_i(s) \quad (30)$$

Rearrangement of Equation 30 yields

$$\delta c_i(s) = \frac{\beta_i k}{p \ell^* e^{-B^2 \tau}} \frac{1}{s + \lambda_i} \delta n(s) \quad (31)$$

ordinary differential equation with its solution the last two cases.

$$(100) \quad \frac{d^2\theta}{dt^2} + \omega_0^2 \sin^2 \theta = \omega_0^2 \left(1 - \frac{\omega_0^2}{\omega^2} \cos^2 \theta \right) = \omega_0^2 \left(1 - \frac{\omega_0^2}{\omega^2} \right) + \frac{\omega_0^2}{\omega^2} \cos^2 \theta$$

Ans

$$(101) \quad (\omega^2 - \omega_0^2) \frac{d^2\theta}{dt^2} + \frac{\omega_0^2 + \omega_0^2}{\omega^2} \frac{d\theta}{dt} = \frac{\omega_0^2}{\omega^2} \times \frac{d\theta}{dt}$$

and "integrating" one "for" and "the" solution. The same will make addition easy. It has been applied to differential "equations" which are not always "readable" and "true" of things they have

$$(102) \quad \frac{d^2\theta}{dt^2} + \omega_0^2 \frac{d\theta}{dt} = \omega^2 - \frac{\omega_0^2}{\omega^2} - \frac{2}{\omega^2} \frac{d\theta}{dt} = \frac{\omega_0^2}{\omega^2}$$

$$(103) \quad \frac{d^2\theta}{dt^2} + \omega_0^2 \frac{d\theta}{dt} = \omega^2 - \frac{\omega_0^2}{\omega^2} = \frac{\omega_0^2}{\omega^2}$$

where the first term is "negligible" making instead of undetermined eq

$$(104) \quad \frac{d^2\theta}{dt^2} + \frac{\omega_0^2}{\omega^2} \sin^2 \theta = \text{const} \frac{d\theta}{dt} - (\text{const}) \frac{d^2\theta}{dt^2} = (\text{const})$$

$$(105) \quad (\text{const})$$

$$(106) \quad (\text{const}) \frac{d\theta}{dt} = (\text{const}) \frac{d^2\theta}{dt^2} = (\text{const})$$

which is "readable" and "true"

$$(107) \quad \frac{d\theta}{dt} = \frac{1}{(\text{const})} \frac{d^2\theta}{dt^2} = (\text{const})$$

Substitution of Equation 31 in Equation 29 and collecting terms yields

$$\mathcal{S}n(s) \left\{ s - \frac{k-1}{\ell^*} + \frac{k\beta}{\ell^*} - \sum_{i=1}^6 \frac{\beta_i \lambda_i k}{\ell^*(s + \lambda_i)} \right\} = \mathcal{S}\epsilon(s) \quad (32)$$

By rearranging Equation 32, the transfer function becomes

$$\frac{\mathcal{S}n(s)}{\mathcal{S}\epsilon(s)} = \frac{\ell^*}{s\ell^* - k + 1 + k\beta - k \sum_{i=1}^6 \frac{\beta_i \lambda_i}{s + \lambda_i}} \quad (33)$$

By substitution of $\frac{\bar{\lambda}\beta}{s + \bar{\lambda}}$ for $\sum_{i=1}^6 \frac{\beta_i \lambda_i}{s + \lambda_i}$ Equation 33 becomes

$$\frac{\mathcal{S}n(s)}{\mathcal{S}\epsilon(s)} = \frac{\ell^*}{s\ell^* - k(1 - \beta) + 1 - k \frac{\bar{\lambda}\beta}{s + \bar{\lambda}}} \quad (34)$$

Multiplication of the numerator and denominator by $(s + \bar{\lambda})$ and division by ℓ^* yields

$$\frac{\mathcal{S}n(s)}{\mathcal{S}\epsilon(s)} = \frac{s + \bar{\lambda}}{s^2 + (\bar{\lambda} + \frac{1-k}{\ell^*} + \frac{k\beta}{\ell^*})s + \frac{\bar{\lambda}}{\ell^*}(1-k)} \quad (35)$$

The denominator is of quadratic form, and when average values of $\bar{\lambda} = 0.08$, $k = 0.5$, $\ell^* = 0.001$, and $\beta = 0.0064$ are substituted in the coefficient of s , it is seen that $\bar{\lambda}$ and $\frac{k\beta}{\ell^*}$ are negligible compared to $\frac{1-k}{\ell^*}$ so that

$$\frac{\mathcal{S}n(s)}{\mathcal{S}\epsilon(s)} = \frac{s + \bar{\lambda}}{s^2 + (\frac{1-k}{\ell^*})s + \bar{\lambda} \frac{(1-k)}{\ell^*}} \quad (36)$$

The roots of the denominator are found by use of the quadratic formula.

and we can find the value of λ which satisfies the condition of the stability analysis.

$$(10) \quad \text{Given } \lambda = \left\{ \frac{\sqrt{1 - \frac{1}{4}k^2}}{2} + i\sqrt{1 - \frac{1}{4}k^2 - k} \right\} \text{ which}$$

is equal to $\lambda = 0.5 + 0.5i$ which corresponds to

$$(11) \quad \frac{\lambda - 0.5}{\lambda - 0.5} = \frac{0.5 + 0.5i}{0.5 - 0.5i} = \frac{1+i}{1-i}$$

which is equivalent to $\lambda = \frac{1+i}{1-i}$ or $\lambda = \frac{1+i}{1-i}$. So substituting λ into

$$(12) \quad \frac{\lambda - 0.5}{\lambda - 0.5} = \frac{1+i}{1-i} = \frac{1+i}{1-i} = \frac{1+i}{1-i}$$

($i = \sqrt{-1}$) we can obtain the equation for the natural frequency ω_0 by solving the

$$(13) \quad \frac{\lambda - 0.5}{\lambda - 0.5} = \frac{1+i}{1-i} = \frac{1+i}{1-i} = \frac{1+i}{1-i}$$

and we can get the result of $\omega_0 = 0.5\sqrt{1 - \frac{1}{4}k^2}$ or $\omega_0 = 0.5\sqrt{1 - \frac{1}{4}k^2}$. So the λ with $\omega_0 = 0.5\sqrt{1 - \frac{1}{4}k^2}$ is the solution of the equation $\lambda = \frac{1+i}{1-i}$ or $\lambda = \frac{1+i}{1-i}$.

$$(14) \quad \frac{\lambda - 0.5}{\lambda - 0.5} = \frac{1+i}{1-i} = \frac{1+i}{1-i} = \frac{1+i}{1-i}$$

and this is the value of λ which corresponds to the eigenvalue of the system and

$$s = \frac{1-k}{2\ell^*} \left\{ -1 \pm \left[1 - \frac{4\bar{\lambda}\ell^*}{1-k} \right]^{\frac{1}{2}} \right\}$$

From the binomial expansion $(a \pm b)^n = a \pm nb \dots$ where $b \ll a$, the radical expands to $\left[1 - \frac{4\bar{\lambda}\ell^*}{1-k} \right]^{\frac{1}{2}} \approx 1 - \frac{2\bar{\lambda}\ell^*}{1-k} \dots$.

By substitution of the expanded terms for the radical, s becomes

$$s \approx \frac{1-k}{2\ell^*} \left\{ -1 \pm \left[1 - \frac{2\bar{\lambda}\ell^*}{1-k} \right] \right\}$$

First root

$$s_1 = \frac{1-k}{2\ell^*} \left\{ -1 + 1 - \frac{2\bar{\lambda}\ell^*}{1-k} \right\} = -\bar{\lambda}$$

Second root

$$s_2 = \frac{1-k}{2\ell^*} \left\{ -1 - 1 + \frac{2\bar{\lambda}\ell^*}{1-k} \right\} = -\left(\frac{1-k}{\ell^*}\right) + \bar{\lambda}$$

When $\bar{\lambda}$ is considered negligible compared to the first term the resultant root is

$$s_2 = -\left(\frac{1-k}{\ell^*}\right)$$

The denominator is now seen to be factorable to a good approximation into

$$(s + \bar{\lambda})(s + \frac{1-k}{\ell^*})$$

By substitution back into Equation 36 the result is

$$\frac{S_n(s)}{S_A(s)} = \frac{s + \bar{\lambda}}{(s + \bar{\lambda})(s + \frac{1-k}{\ell^*})} = \frac{1}{s + \frac{1-k}{\ell^*}} \quad (37)$$

This is the final form of the transfer function. It shows that for a subcritical assembly one may ignore the effect of

where $\lambda = \frac{1}{2}(\mu_1 + \mu_2)$ and μ_1, μ_2 are the eigenvalues of A . Then $\lambda^2 - \mu_1\lambda + \mu_1\mu_2 = 0$ has roots $\frac{\lambda - \mu_1}{2} \pm \sqrt{\frac{(\lambda - \mu_1)^2}{4} - \mu_1\mu_2}$ and since $\lambda > \mu_1$ it follows that $\lambda - \mu_1 > 0$ so the two roots are real and distinct. It follows that the eigenvalues of A are $\lambda \pm \sqrt{\lambda^2 - \mu_1\lambda + \mu_1\mu_2}$.

$$\left(\left[\frac{\lambda^2 - \mu_1\lambda + \mu_1\mu_2}{2} \right] \pm \sqrt{\frac{(\lambda - \mu_1)^2}{4} - \mu_1\mu_2} \right) \text{diag}(B).$$

Now note

$$\left(\left[\frac{\lambda^2 - \mu_1\lambda + \mu_1\mu_2}{2} \right] \pm \sqrt{\frac{(\lambda - \mu_1)^2}{4} - \mu_1\mu_2} \right) \text{diag}(B) =$$

now apply

$$\left(\lambda + (\lambda^2 - \mu_1\lambda + \mu_1\mu_2) \right) \pm \sqrt{\frac{(\lambda - \mu_1)^2}{4} - \mu_1\mu_2} \text{diag}(B) = g^2.$$

Thus $\det(g^2 - A) = \det((\lambda + (\lambda^2 - \mu_1\lambda + \mu_1\mu_2)) \text{diag}(B)) = 0$ since λ is an eigenvalue of A .

$$\det(g^2 - A) = \det((\lambda + (\lambda^2 - \mu_1\lambda + \mu_1\mu_2)) \text{diag}(B)) = 0$$

so λ is an eigenvalue of $(g^2 - A)$ or λ is an eigenvalue of $(A^2 - \mu_1A + \mu_1\mu_2I)$.

$$\det((\lambda + (\lambda^2 - \mu_1\lambda + \mu_1\mu_2)) \text{diag}(B)) = 0$$

so λ is an eigenvalue of $(A^2 - \mu_1A + \mu_1\mu_2I)$.

$$(A^2 - \mu_1A + \mu_1\mu_2I) = \frac{\lambda^2 - \mu_1\lambda + \mu_1\mu_2}{\lambda - \mu_1} \text{diag}(B)$$

so all eigenvalues of $(A^2 - \mu_1A + \mu_1\mu_2I)$ are λ and $\lambda - \mu_1$. Since $\lambda > \mu_1$ it follows that $\lambda - \mu_1 < \lambda$ so the eigenvalues of $(A^2 - \mu_1A + \mu_1\mu_2I)$ are λ and $\lambda - \mu_1$.

the delayed neutrons in determining the change in neutron population due to a variation in the source. The terms containing the average delayed neutron decay constant cancel out, and thus the prompt neutron lifetime can be replaced by the average neutron lifetime. The assumption that the effect of the delayed neutrons can be ignored is valid for k_{eff} 0.995 (1). The subcritical assembly utilized in this investigation has a k_{eff} of about 0.52, hence the above assumption should be valid. The same result, Equation 37, is obtained from Equations 17 and 18 when the delayed neutron terms containing $\lambda_i c_i$ are deleted and the same derivation procedure as above followed.

Transfer Function with Reactivity Variation

Again utilizing Equation 17 and Equation 18 for the transfer function and Equations 22 and 23 for the steady state condition, let a small variation δk be imposed upon k so that

$$\begin{aligned} k &= k_0 + \delta k \\ n &= n_0 + \delta n \\ c_i &= c_{i0} + \delta c_i \end{aligned} \tag{38}$$

where δk is the amplitude of the variation, and not the instantaneous value of the excess reactivity. Substitution of Equation 38 into Equations 17 and 18 yields

$$\frac{dn}{dt} = \frac{d\mathcal{S}_n}{dt} = \left[(k_o + \mathcal{S}_k)(1 - \beta) - 1 \right] \frac{(n_o + \mathcal{S}_n)}{\ell^*} + pe^{-B^2\tau} \sum_{i=1}^6 \lambda_i (c_{i_o} + \mathcal{S}_{c_i}) + \mathcal{A} \quad (39)$$

$$\frac{dc_i}{dt} = \frac{d\mathcal{S}_{c_i}}{dt} = \beta_i \frac{(k_o - \mathcal{S}_k)(n_o + \mathcal{S}_n)}{p \ell^* e^{-B^2\tau}} - \lambda_i (c_{i_o} + \mathcal{S}_{c_i}) \quad (40)$$

By multiplication, setting products of small numbers equal to zero, and collecting the steady state terms of Equations 22 and 23 and equating them to zero one obtains

$$\begin{aligned} \frac{d\mathcal{S}_n}{dt} &= \frac{n_o \mathcal{S}_k}{\ell^*} + \frac{k_o \mathcal{S}_n}{\ell^*} - \beta \frac{k_o \mathcal{S}_n}{\ell^*} - \beta \frac{n_o \mathcal{S}_k}{\ell^*} \\ &- \frac{\mathcal{S}_n}{\ell^*} + pe^{-B^2\tau} \sum_{i=1}^6 \lambda_i \mathcal{S}_{c_i} \end{aligned} \quad (41)$$

$$\frac{d\mathcal{S}_{c_i}}{dt} = \frac{\beta_i k_o \mathcal{S}_n}{p \ell^* e^{-B^2\tau}} + \frac{\beta_i n_o \mathcal{S}_k}{p \ell^* e^{-B^2\tau}} - \lambda_i \mathcal{S}_{c_i} \quad (42)$$

By transformation to Laplace notation the equations become

$$\begin{aligned} s\mathcal{S}_n(s) &= \frac{n_o}{\ell^*} \mathcal{S}_k(s) + \frac{k_o}{\ell^*} \mathcal{S}_n(s) - \frac{\beta k_o}{\ell^*} \mathcal{S}_n(s) - \frac{\beta n_o}{\ell^*} \mathcal{S}_k(s) \\ &- \frac{\mathcal{S}_n(s)}{\ell^*} + pe^{-B^2\tau} \sum_{i=1}^6 \lambda_i \mathcal{S}_{c_i}(s) \end{aligned} \quad (43)$$

$$\mathcal{S}_{c_i}(s) = \frac{\beta_i k_o}{p \ell^* e^{-B^2\tau}} \mathcal{S}_n(s) + \frac{\beta_i n_o}{p \ell^* e^{-B^2\tau}} \mathcal{S}_k(s) - \lambda_i \mathcal{S}_{c_i}(s) \quad (44)$$

Solution of Equation 44 for $\mathcal{S}_{c_i}(s)$, substitution in Equation 43, and rearranging yields the transfer function

the same time, the following question arises:

(iii) What is the value of $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right)$?

$$(iii) \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \dots = \frac{\infty}{2}$$

From the above discussion, we can say that the function $f(x) = \frac{1}{x}$ is discontinuous at every rational number and continuous at every irrational number. And also the function $f(x) = \frac{1}{x}$ is discontinuous at every rational number and continuous at every irrational number.

$$\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots = \frac{\infty}{2}$$

$$(iv) \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \frac{1}{2} + \frac{1}{2} + \dots = \frac{\infty}{2}$$

$$(v) \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \frac{1}{2} + \frac{1}{2} + \dots = \frac{\infty}{2}$$

From the above two examples, we can say that the function $f(x) = \frac{1}{x}$ is discontinuous at every rational number and continuous at every irrational number.

$$(vi) \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \frac{1}{2} + \frac{1}{2} + \dots = \frac{\infty}{2}$$

$$(vii) \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \frac{1}{2} + \frac{1}{2} + \dots = \frac{\infty}{2}$$

$$(viii) \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f\left(\frac{k}{n}\right) = \frac{1}{2} + \frac{1}{2} + \dots = \frac{\infty}{2}$$

From the above two examples, we can say that the function $f(x) = \frac{1}{x}$ is discontinuous at every rational number and continuous at every irrational number. And also the function $f(x) = \frac{1}{x}$ is discontinuous at every rational number and continuous at every irrational number.

$$\frac{\mathcal{S}_n(s)}{\mathcal{S}_k(s)} = \frac{n_o}{k_o} \frac{1 - \beta + \sum_{i=1}^6 \frac{\lambda_i \beta_i}{s + \lambda_i}}{\frac{s\ell^*}{k_o} + \frac{1}{k_o} - 1 + \beta - \sum_{i=1}^6 \frac{\lambda_i \beta_i}{s + \lambda_i}} \quad (45)$$

The substitution of $\frac{\bar{\lambda}\beta}{s + \bar{\lambda}}$ for $\sum_{i=1}^6 \frac{\lambda_i \beta_i}{s + \lambda_i}$ in Equation 45 yields

$$\frac{\mathcal{S}_n(s)}{\mathcal{S}_k(s)} = \frac{n_o}{k_o} \cdot \frac{1 - \beta + \frac{\bar{\lambda}\beta}{s + \bar{\lambda}}}{\frac{s\ell^*}{k_o} + \frac{1}{k_o} - 1 + \beta - \frac{\bar{\lambda}\beta}{s + \bar{\lambda}}} \quad (46)$$

The multiplication of numerator and denominator by $s + \bar{\lambda}$ and collection of terms yields

$$\frac{\mathcal{S}_n(s)}{\mathcal{S}_k(s)} = \frac{n_o}{k_o} \cdot \frac{s(1 - \beta) + \bar{\lambda}}{s^2(\frac{\ell^*}{k_o}) + (\frac{\bar{\lambda}}{k_o} + \frac{1-k_o}{k_o} + \beta)s + \bar{\lambda}(\frac{1-k_o}{k_o})} \quad (47)$$

Division of the numerator and denominator by ℓ^*/k_o yields

$$\frac{\mathcal{S}_n(s)}{\mathcal{S}_k(s)} = \frac{n_o}{k_o} \frac{\frac{k_o}{\ell^*} \{s(1 - \beta) + \bar{\lambda}\}}{s^2 + (\bar{\lambda} + \frac{1-k_o}{\ell^*} + \frac{\beta k_o}{\ell^*})s + \bar{\lambda}(\frac{1-k_o}{\ell^*})} \quad (48)$$

This equation is seen to have the same denominator as Equation 35 which simplifies and factors to $(s + \bar{\lambda})(s + \frac{1-k_o}{\ell^*})$. In the numerator the factor $(1 - \beta)$ is seen to be nearly equal to one when 0.0064 is substituted for β . Equation 48 then becomes

$$\frac{\mathcal{S}_n(s)}{\mathcal{S}_k(s)} = \frac{n_o/\ell^*}{s + \frac{1-k_o}{\ell^*}} \quad (49)$$

which is the final form of the transfer function. This result

$$(14) \quad \frac{\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) + \lambda - x}{\lambda^2 - x^2} \cdot \frac{x^2}{x^2 - \lambda^2} = \frac{\lambda^2 - x^2}{\lambda^2 - x^2}$$

Die Gleichung ist $\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) + \lambda - x = 0$ zu untersuchen bzW
nach x.

$$(15) \quad \frac{\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) + \lambda - x}{\lambda^2 - x^2} \cdot \frac{x^2}{x^2 - \lambda^2} = \frac{\lambda^2 - x^2}{\lambda^2 - x^2}$$

Die Gleichung ist $\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) + \lambda - x = 0$ zu untersuchen bzW
nach x.

$$(16) \quad \frac{\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) + \lambda - x}{\lambda^2 - x^2} \cdot \frac{x^2}{x^2 - \lambda^2} = \frac{\lambda^2 - x^2}{\lambda^2 - x^2}$$

Zulässig $x \neq \lambda$ zu untersuchen die Gleichung ist $\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) + \lambda - x = 0$

$$(17) \quad \frac{\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) + \lambda - x}{\lambda^2 - x^2} \cdot \frac{x^2}{x^2 - \lambda^2} = \frac{\lambda^2 - x^2}{\lambda^2 - x^2}$$

Unter den Bedingungen dass $x \neq \lambda$ und $\lambda \neq 0$ muss die Gleichung $\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) + \lambda - x = 0$ untersucht werden. Dazu ist die Ableitung von $\frac{1}{\lambda^2 - x^2}$ zu bestimmen. Es gilt $\frac{d}{dx} \left(\frac{1}{\lambda^2 - x^2} \right) = \frac{2x}{(\lambda^2 - x^2)^2}$. Die Gleichung ist $\frac{2x}{(\lambda^2 - x^2)^2} + \lambda - x = 0$ zu untersuchen bzW nach x.

$$\frac{\frac{2x}{(\lambda^2 - x^2)^2} + \lambda - x}{\lambda^2 - x^2} \cdot \frac{x^2}{x^2 - \lambda^2} = \frac{\lambda^2 - x^2}{\lambda^2 - x^2}$$

Dann ist $\frac{2x}{(\lambda^2 - x^2)^2} + \lambda - x = 0$ zu untersuchen bzW nach x.

is the same as Equation 32 where the source strength was varied except for the constant multiplier n_0/ℓ^* . The average decay constant $\bar{\lambda}$ and fraction of delayed neutrons drop out of the final expression as before, showing that the transfer function for a subcritical assembly is independent of the delayed neutrons and dependent on the average neutron lifetime.

Modulus of the Transfer Function

Division of the transfer function, Equation 49, by $\frac{1-k_o}{\ell}$ and substitution of $S = j\omega$, yields

$$\frac{\mathcal{S}n(s)}{\mathcal{S}k(s)} = \frac{n_o/1-k_o}{1 + \frac{j\omega\ell}{1-k_o}} \quad (50)$$

The result of normalizing Equation 50 and setting it equal to $Y(j\omega)$ is

$$Y(j\omega) = \frac{\mathcal{S}n(s)/n_o}{\mathcal{S}k(s)/1-k_o} = \frac{1}{1 + \frac{j\omega\ell}{1-k_o}} \quad (51)$$

It is seen from Equation 51 that the prompt neutron break frequency, ω_p , is equal to $\frac{1-k}{\ell}$ for a subcritical assembly. ω_p effectively determines k for the subcritical assembly if ℓ is known, and the reverse is also true. The numerator and denominator are multiplied by the complex conjugate, $1 - \frac{j\omega\ell}{1-k_o}$, yielding

was measured using only about 25 minutes of time and a
series of 1000 individual frames and no prior belief
in any particular model or parameter. A measure which
reflects our prior beliefs about an unknown truth will be
the "posterior" of θ (where θ is the true value) will be
with certain elements of the posterior law remaining identical

with

obtained without any data.

$\frac{d\pi}{d\theta}$ of $P(\theta)$ obtained without coherent prior knowledge
is given by $\pi(\theta) = 1$ for $0 < \theta < 1$

$$\text{After } n \text{ observations } - \text{observed values } \theta_1, \theta_2, \dots, \theta_n \text{ and } \theta_i \sim \frac{\text{prior}}{\text{posterior}} = \frac{\pi(\theta)}{\pi(\theta|D)}$$

and $P(\theta|D)$ can be obtained from the Bayes rule
as $P(\theta|D) \propto P(D|\theta)P(\theta)$

$$(17) \quad \frac{P(\theta|D)}{P(\theta)} = \frac{P(D|\theta)\pi(\theta)}{P(D|\theta)\pi(\theta)} = \left(\frac{D}{n} \right)^n$$

After n random samples are used it follows that each of the
observed distributions is not $\frac{1}{n}$ of some θ_{true} with uncertainty
of θ_{true} . Uncertainty will not be uniform around the
posterior and will also be greater for low prior
uncertainties whereas for high prior uncertainties the

$$\text{posterior } \frac{1}{n} = 1$$

and so the uncertainty around the true value will be

$$Y(j\omega) = \frac{1 - \frac{j\omega\ell}{1-k_0}}{1 + (\frac{\omega\ell}{1-k_0})^2} \quad (52)$$

The modulus of the transfer function, or the magnitude of $Y(j\omega)$, is

$$|Y(j\omega)| = \frac{\left\{1^2 + \left(\frac{\omega\ell}{1-k_0}\right)^2\right\}^{\frac{1}{2}}}{1 + \left(\frac{\omega\ell}{1-k_0}\right)^2}$$

which reduces to

$$|Y(j\omega)| = \frac{1}{\left\{1 + \left(\frac{\omega\ell}{1-k_0}\right)^2\right\}^{\frac{1}{2}}} \quad (53)$$

The square of the modulus of the transfer function $|Y(j\omega)|^2$ is then

$$|Y(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega\ell}{1-k_0}\right)^2} \quad (54)$$

The square of the modulus of the transfer function is the basis upon which the parameters of the transfer function can be evaluated as will be shown in the next section.

$$\text{102} \quad \frac{\partial}{\partial z} \left(\frac{\partial \omega_1}{\partial z} \right) = \frac{\partial^2 \omega_1}{\partial z^2} \text{ and } \frac{\partial^2 \omega_1}{\partial z^2} < 0$$

- Derivatives with respect to ω_1 and ω_2 are zero at $(\bar{\omega}_1, \bar{\omega}_2)$

$$\text{103} \quad \frac{\partial^2 \omega_1}{\partial z^2} = \frac{\partial^2 \omega_1}{\partial z^2} = \frac{\partial^2 \omega_1}{\partial z^2} > 0$$

- Second derivatives are positive definite

$$\text{104} \quad \frac{\partial^2 \omega_1}{\partial z^2} = \frac{\partial^2 \omega_1}{\partial z^2} = \frac{\partial^2 \omega_1}{\partial z^2} > 0$$

$\Rightarrow \{\omega_1, \omega_2\}$ without boundary has no extreme and

$$\text{105} \quad \frac{\partial^2 \omega_1}{\partial z^2} = \frac{\partial^2 \omega_1}{\partial z^2} = \frac{\partial^2 \omega_1}{\partial z^2} > 0$$

- At boundary values of ω_1 extreme will be unique and

- boundary values are decreasing and outside may also

- decrease, thus not a point of little or intention of

- boundary values are decreasing and outside may also

- decrease, thus not a point of little or intention of

STOCHASTIC EVALUATION OF THE TRANSFER FUNCTION

The transfer function of an assembly can be determined experimentally by introducing sinusoidal variations in the input and measuring the phase angle and amplitude attenuation with frequency of the resulting sinusoidal variation in the output. This is usually accomplished by sinusoidally varying the strength of the neutron source or the reactivity and noting the resulting variation in the neutron flux. Variations in the neutron population due to the statistical nature of the fission process cause a change in reactivity and a fluctuation of the power level of a critical assembly. These random variations in the flux level may be considered as a summation of Fourier series which transform into Fourier integrals when the period approaches infinity (2). The transfer function of an assembly could then logically be extended to represent the response of a system to a random as well as sinusoidal input.

Consider a voltage which is a random or stochastic function of time and limited to a small range of frequencies between f_1 and f_2 . One can determine a value which is proportional to the average power by squaring the function, integrating, and dividing the result by the time over which the integration was performed. If this "average power" is divided by $\Delta f = f_1 - f_2$, the frequency increment which the average power represents, the result is the average power or mean

ИССЛЕДОВАНИЯ НАУЧНОГО СОВЕТА ПО АСТРОНОМИИ

Исследования по астрономии включают изучение звездного неба и его движений, звездных явлений и явлений гравитации, а также изучение звездных систем и космических объектов, включая галактики, звездные скопления, кометы и т. д.

Большое внимание уделяется изучению звездного неба и его движений, а также изучению звездных явлений и явлений гравитации.

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square voltage (amplitude) per unit frequency. This is called the power spectral density function. The term power spectral density function has been extended to apply to any function which consists of mean square signal amplitude per unit frequency of the band width investigated.

Nuclear reactions and fission processes which give rise to the neutron flux in a subcritical assembly can be considered as a random variable which has a normal or Gaussian distribution about some mean value of frequency of occurrence. The power spectral density of such a random variable is the average power (mean square signal amplitude) per unit frequency of the band width measured (2). It has the unique property of being a constant for this type of function and is defined as a "white noise". When such a random variable is passed through a linear system the output is a random variable which has been attenuated by the system characteristics (10).

If a system such as the subcritical assembly is excited by a white noise, which has a constant amplitude at all frequencies, one can analyze the output of the assembly over a range of frequencies to determine the response or transfer function (8). It has been shown mathematically that the output power density spectrum of a chain reacting system is proportional to the square of the modulus of the transfer function (11). This has also been demonstrated experimentally

the first year and the corresponding figure for 1998 was 1.1 million. This means that the number of foreign tourists increased by 1.1 million. The growth in foreign tourists is due to the following factors:
- increasing availability of foreign currency and increasing stability of the exchange rate;
- increasing availability of foreign currency and the growth in the number of foreign tourists.

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for several nuclear reactors (4, 8).

Figure 1 shows an ideal system for power spectral density measurement in which it is assumed that the electronic components have perfect frequency response over the range of the investigation and introduce no noise. Under these conditions the system equation would be

$$G_{yy}(\omega) = G_{xx}(\omega) |Y(j\omega)|^2 = K |Y(j\omega)|^2 \quad (55)$$

where $G_{xx}(\omega)$ = input power spectral density = constant.

$G_{yy}(\omega)$ = output power spectral density.

$Y(j\omega)$ = system transfer function.

$|Y(j\omega)|^2$ = square of modulus of transfer function.

Since the frequency response of the measuring equipment is not perfect, it also introduces a transfer function into the equation. This may be neglected and the response of the measuring equipment considered independent of frequency, if the break frequency of the equipment is beyond the range being investigated (8). The final form of the system equation was shown to be (8)

$$G_{yy}(\omega) = A + B |Y(j\omega)|^2 \quad (56)$$

where A represents the level of measuring equipment noise and B represents the level of the assembly noise. The substitution of Equation 54 into Equation 56 yields

the same time, the number of the anti-pedestrian measures seems to have increased. In 2000, 10% of the 100 most populated municipalities had at least one anti-pedestrian measure, while in 2009, 25% had at least one. The report also quantifies the number of different anti-pedestrian measures. It shows several trends: systematic reduction of pedestrian areas and the removal of pedestrian crossings, and many anti-pedestrian measures with unclear effects.

Table 1 shows a detailed overview of the changes in the 100 most populated municipalities between 2000 and 2009. The table shows the number of different measures taken in each municipality, the percentage of the population living in areas with anti-pedestrian measures, and the percentage of the population living in areas with unclear effects. The table also shows the percentage of the population living in areas with anti-pedestrian measures that have been reduced or removed since 2000. The table shows that the percentage of the population living in areas with anti-pedestrian measures has increased from 10% in 2000 to 25% in 2009. The percentage of the population living in areas with unclear effects has decreased from 80% in 2000 to 75% in 2009. The percentage of the population living in areas with anti-pedestrian measures that have been reduced or removed has decreased from 10% in 2000 to 5% in 2009.

The table also shows the percentage of the population living in areas with anti-pedestrian measures that have been reduced or removed. This percentage has increased from 10% in 2000 to 20% in 2009. The table also shows the percentage of the population living in areas with unclear effects that have been reduced or removed. This percentage has decreased from 80% in 2000 to 60% in 2009.



Figure 1. Ideal system for determination of output power spectral density function

$$G_{yy}(\omega) = A + \frac{B}{1 + (\frac{\omega l}{1-k})^2} \quad (57)$$

A sketch of Equation 57 is shown in Figure 2.

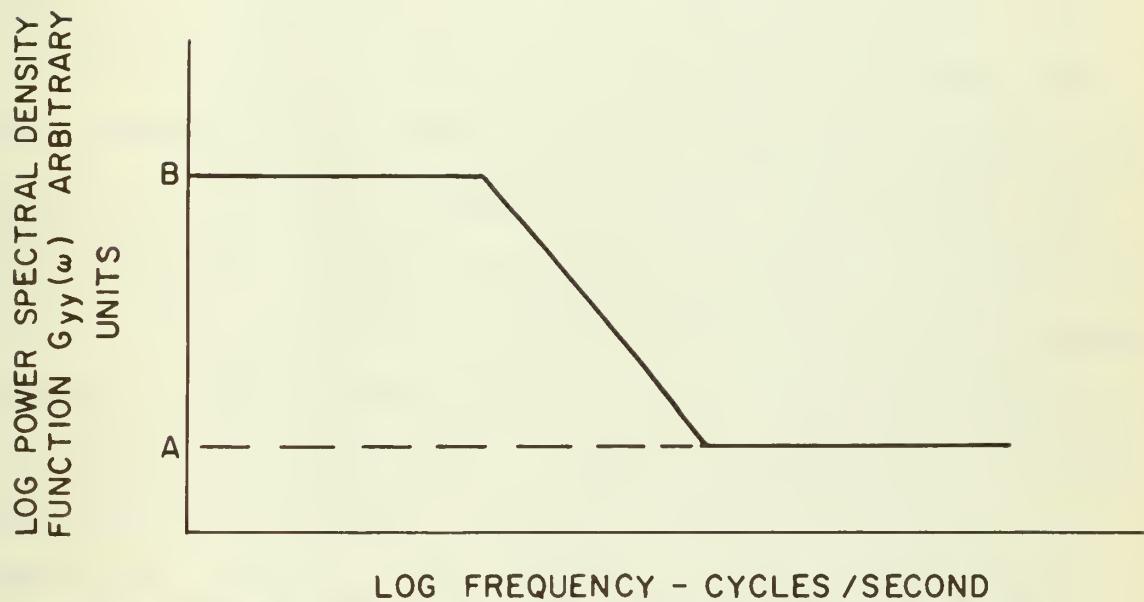


Figure 2. Ideal power spectral density function of an assembly

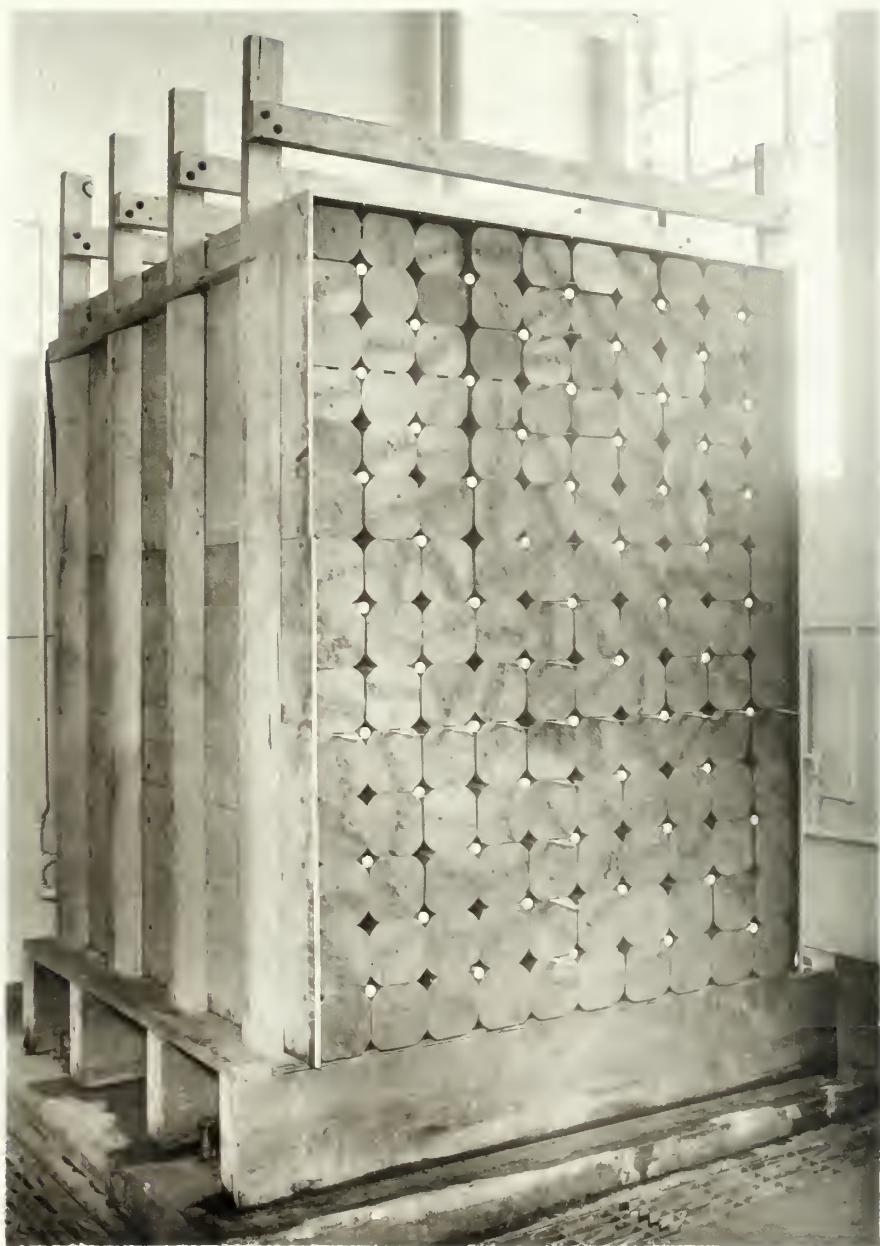
DESCRIPTION OF APPARATUS AND EXPERIMENTAL PROCEDURE

The graphite moderated subcritical assembly (^{14}N) utilized in this investigation was constructed at Iowa State University in 1957. Figure 3 is a photograph of the subcritical assembly with the end cover removed. The blocks were constructed from AGR grade graphite cylinders with diameters of 7.0 in. and 6 3/8 in., and length of 60 in. The lower nine layers were made up of blocks with a 6.0 inch square cross section and rounded corners of radius 3.5 inches. The top five layers were constructed of blocks with a 6.0 inch by 5.0 inch cross section with rounded corners of radius 3 3/16 inches. The density of the graphite was 1.56 gm. per cu. cm. or 97.3 lb. per cu. ft. An 8 $\frac{1}{2}$ inch lattice was formed by placing uranium slugs in alternate channels as shown in Figure 3. The slugs of uranium were 1.000 in. in diameter and 8.00 in. in length and encased in aluminum cylinders 0.040 in. thick and with ends 0.20 in. thick. The outside dimensions were 1.080 in. in diameter and 6.40 in. in length. The uranium density was 19.0 gm. per cu. cm. or 1186 lb. per cu. ft.

The assembly was covered on the top and sides with a 0.010 in. sheet of cadmium sandwiched between a 3/8 in. sheet of plywood and a 1/8 in. sheet of masonite. An effective "black wall" for thermal neutrons was produced by the cadmium. The horizontal distances between the side sheets of cadmium was 60.5 in. and 62.5 in. With the extrapolation lengths

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Figure 3. Front view of the subcritical assembly



added, the effective size of the assembly was approximately 62 in. (side to side) by 64 in. (front to back).

The subcritical assembly was mounted on a pedestal as shown in Figure 3. The three spaces under the assembly housed 11.5 in. deep water tanks. The water provided protection from the neutron source which was mounted in the center tank under the pile.

The neutron source consisted of five individual plutonium-beryllium sources in containers 1 in. in diameter and 1 3/8 in. in length. The individual source strength was rated at one curie emitting approximately 1.63×10^6 neutrons per sec. The total source strength was 8.15×10^6 neutrons per sec. The five sources were mounted on a pedestal in the center tank which was filled with water. The sources were located on the centerline of the assembly directly below the bottom layer of graphite. Five holes were cut along the center line of the front panel of the assembly at one foot vertical intervals. The holes were cut to correspond to vacant channels in the lattice for insertion of the BF_3 neutron detector. The lowest hole, one foot above the bottom of the assembly, will be referred to as hole one, the hole two feet in height as hole two and so on.

The electronic circuitry for measuring the random variations in the neutron flux is shown in Figure 4. The neutron probe used was a Wood Counter Laboratory BF_3 detector. A

of his experience was different with the wife and family and friends.
Indeed we could see the depth of their love and the
true fulfillment in the marriage was different. Different in
depth, richness and more personal needs and character of work
and family. Indeed, there are varied relationships and
more richness with the husband and wife, deeper bonds and more
affection with each other.

Relationships with the children remain similar and
different at the same time. Children will always be influenced by the
the spiritual values transmitted with integrity and love in the
marriage. $\Delta H = 10.1$ indicates greater stability when the father
absent. $\Delta H = 20.8$ and $\Delta H = 20.9$ indicate lower levels of love and
intimacy in the absence of the husband and wife and
when absent. When the wife is absent there is less intimacy and
less support especially between the wife and husband and no emotional
ties made with many others with integrity in typical married
families. The children will be forced to look with less values
of cooperation in the new union with a previous history
that can be judgmental and critical and at times dominant
and angry and even harsh and cold toward one another and
others and find it as the leaders of this relationship will be
seen as less and also as rigid as don't eat
certain foods and punishment for certain misbehaviors and
punishments will be harsher as much as will continue with the small
and constant ΔH present in the family over a long time.

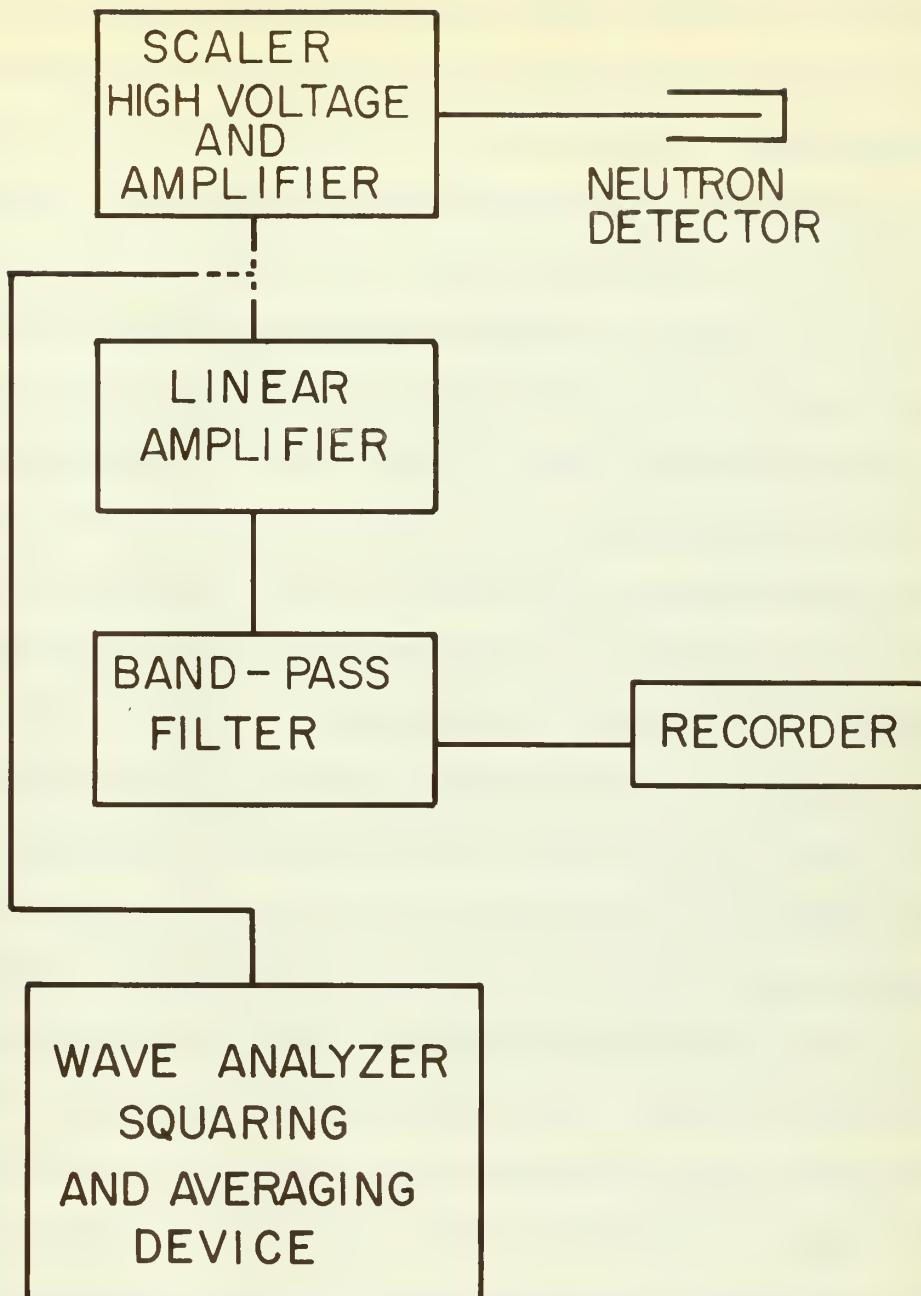


Figure 4. Electronic circuitry

Nuclear Chicago Ultrasclar Model 192A was used to supply high voltage to the detector and to amplify the signal from the detector. A Tektronix Type 112 Direct Coupled Amplifier with a flat frequency response from DC to 2 megacycles was used to further amplify the signal which was fed into a Khron-Hite ultra low frequency Band-Pass Filter, Model 330A. The band width of the filter was adjustable between a low cut off frequency of 0.02 cps to a high cut off frequency of 2000 cps. Frequency adjustment was accomplished by means of two dials calibrated with a logarithmic scale reading directly in cps from 2 to 20 and multiplying switches of 1/100, 1/10, 1, 10 and 100. A capacitance filter in the input circuit filtered out any DC component of the signal. The filter comprised an extremely low noise circuit, with internally generated hum and noise less than 100 microvolts. The filtered random signal was then fed into a two channel Brush Recorder Mark II for further amplification and recording. The recording pen sensitivity was variable from 0.01 to 10 volts per chart line, and the chart speed was variable from one to 125 mm per sec. This circuitry was used to investigate the frequency range of 0.02 cps to 20 cps. An alternate system was used for the frequency range of 20 cps to 100 cps and beyond, to facilitate data taking and processing. All of the components after the Nuclear Chicago Ultrasclar were replaced with a Hewlett-Packard Wave analyzer

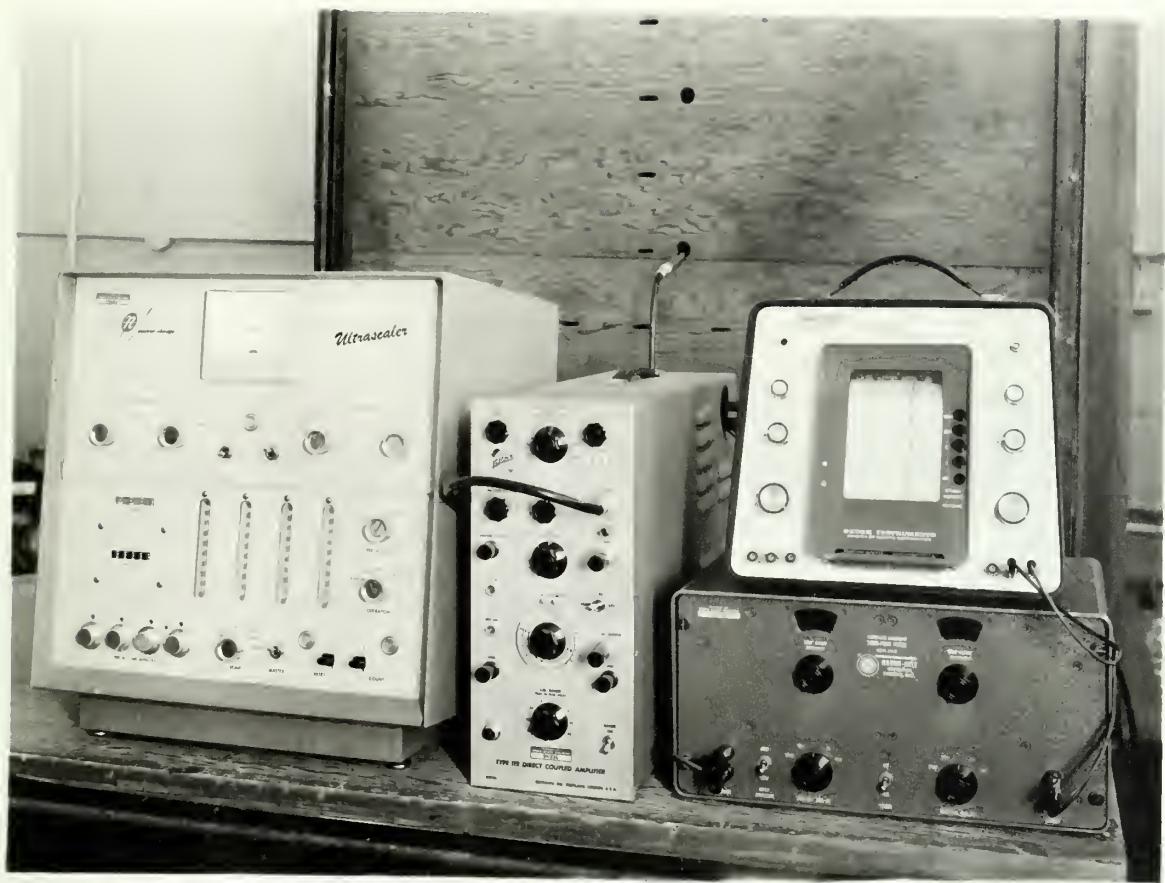
Model 300-1. The wave analyzer had an adjustable band pass of 30 cps to 145 cps, and a frequency range adjustable from zero cps to 16 ke. The output meter was graduated in RMS millivolts. Figure 5 is a photograph of the equipment described above.

Data were taken in each of the five holes, with the probe on the centerline of the assembly. The frequency range investigated was 0.02 cps to 100 cps. The circuitry including the band pass filter and recorder was used for the range of 0.02 cps to 20 cps. The wave analyzer was used from 20 cps to 100 cps. In order to minimize the introduction of spurious signals into the system, the output was taken from the scalar without the counting circuit being activated. The signal was then fed into the Tektronix Type 112 amplifier where the input and output signal was checked periodically with an oscilloscope to insure minimum a.c. pickup and no saturation of the amplifier. The signal was then passed through the band pass filter at a constant per cent band pass such as 0.02 cps to 0.04 cps, 0.04 cps to 0.08 cps, 0.08 cps to 0.16 cps, etc. The output of the band pass filter was recorded on both channels of the Brush recorder for two minutes for each band pass width setting. Typical random data for representative band pass widths are shown in Figure 6.

The power spectral density function for the 0.02 cps to 20 cps range was obtained for each band pass width by a method

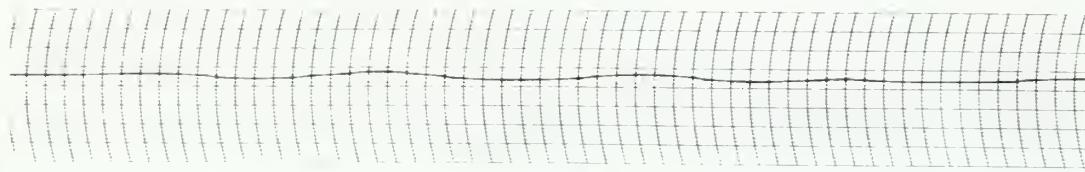
Journal of Clinical Endocrinology 120: 359–367 © 1993 Blackwell Science Ltd

Figure 5. Electronic equipment



new-born and immature cells via some kind of mechanism.

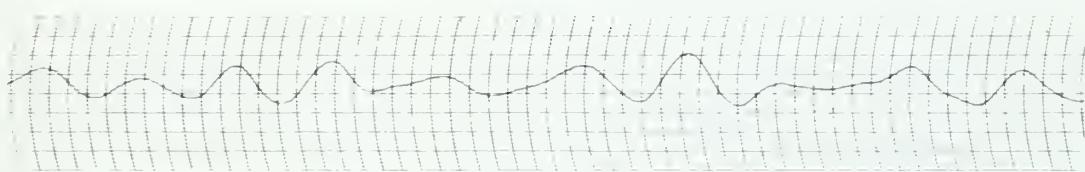
Figure 6. Typical random data for representative band-pass widths



$\Delta f = 0.025 \text{ cps}$
 $\bar{f} = 0.0375 \text{ cps}$



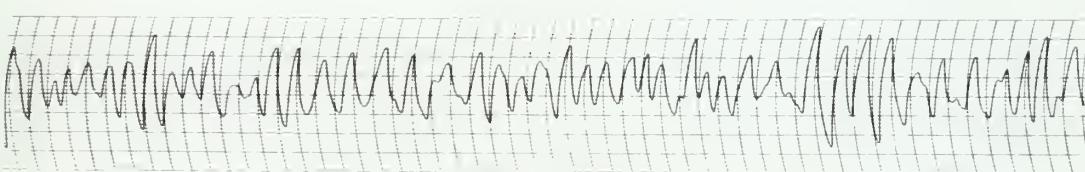
$\Delta f = 0.05 \text{ cps}$
 $\bar{f} = 0.075 \text{ cps}$



$\Delta f = 0.10 \text{ cps}$
 $\bar{f} = 0.15 \text{ cps}$



$\Delta f = 0.20 \text{ cps}$
 $\bar{f} = 0.30 \text{ cps}$



$\Delta f = 0.40 \text{ cps}$
 $\bar{f} = 0.60 \text{ cps}$

All chart speeds 5 mm per sec

$\Delta t = 1.0 \text{ sec}$

similar to that outlined by Murphy (12). One hundred ordinates of each random trace were measured at one second intervals. Each ordinate was squared, and then the ordinates and their squares were summed separately and averaged. The variance, σ^2 , was obtained by taking the difference between the average of the squared value and the square of the average value. This is equivalent to the average of the square of the difference from the mean.

$$\sigma^2 = (\bar{y^2} - \bar{y}^2) = \overline{(y - \bar{y})^2}$$

The power spectral density function $G_{yy}(\omega)$ was then obtained by dividing the variance by the band pass width in cps. This resulted in a value of mean square amplitude per unit frequency, or mean power per unit frequency, for the band width investigated.

$$G_{yy}(\omega) = \frac{\sigma_i^2}{\Delta f_i} \quad \text{mean square amplitude} \\ \text{unit frequency}$$

The power spectral density for the 20 cps to 100 cps range was obtained by squaring the reading from the wave analyzer output meter. The meter read directly in RMS power per unit frequency of the band width investigated. By squaring the reading, the mean square amplitude or power per unit frequency of the band width investigated was obtained.

A sample calculation of the type used to reduce the data from the Brush recorder is included in the Appendix.

$$\langle \hat{v}_x - v_x \rangle = \langle \hat{v}_{y\perp} + \hat{v}_z \rangle = 0$$

二〇

DISCUSSION OF RESULTS

The results of the data reduction are presented graphically in Figures 7 and 8 with the power spectral density, $G_{yy}(\omega)$, in arbitrary units versus frequency in cycles per second. The data were plotted only from 0.03 cps to 10.0 cps because the power spectral density appeared to become constant by 10.0 cps and remained a constant from 10.0 cps to 100 cps. This constant level represents the "A" term in Equation 57 which is the white noise of the instrumentation which is due primarily to the BF_3 detection chamber. The instrumentation noise decreases with increasing height in the assembly. This is explained by the fact that most of the instrumentation noise is due to the flux level or counting rate in the detection chamber which decreases with increasing height.

Figure 7 shows the change in power spectral density function along the axis of the assembly. For hole one ($z = 1$ ft) the power spectral density was determined to be approximately a constant throughout the range of frequencies investigated. This indicated that the random variations in the neutron flux were primarily due to the flux from the source which is "white noise". Hole two ($z = 2$ ft) shows a deviation from the constant power spectral density function of the instrumentation and indicates an increasing contribution from the fission flux and external source which have

theoretical and experimental studies have been made on the influence of the physical environment on the development of the plant. The results of these studies have shown that the physical environment influences the growth and development of the plant in many ways. The physical environment includes temperature, light, water, air, soil, and other factors. Temperature is one of the most important factors that affect the growth and development of the plant. Light is another factor that affects the growth and development of the plant. Water is also an important factor that affects the growth and development of the plant. Air is another factor that affects the growth and development of the plant. Soil is another factor that affects the growth and development of the plant. Other factors such as humidity, wind, and sunlight also affect the growth and development of the plant. The physical environment can be controlled by various methods such as irrigation, fertilization, pruning, and grafting. These methods help to create a favorable environment for the plant to grow and develop. The physical environment can also be improved by selecting suitable varieties of plants that are adapted to the specific conditions of the environment. This can be done by breeding programs that select plants with desirable traits. The physical environment can also be improved by using modern agricultural techniques such as precision agriculture, which involves the use of sensors and computers to monitor and control the growth and development of the plant. This can be done by using remote sensing technology, which involves the use of satellites and drones to monitor the growth and development of the plant. The physical environment can also be improved by using biotechnology, which involves the use of genetic engineering to modify the plant's genes to produce desired traits. This can be done by using recombinant DNA technology, which involves the use of enzymes and other biological agents to alter the plant's genes. The physical environment can also be improved by using organic farming methods, which involve the use of natural fertilizers and pesticides to maintain the health of the plant. This can be done by using compost, manure, and other natural materials to enrich the soil. The physical environment can also be improved by using integrated pest management, which involves the use of natural predators and other methods to control pests without harming the plant. This can be done by using biological control methods, which involve the use of beneficial insects and microorganisms to control pests. The physical environment can also be improved by using agroforestry, which involves the use of trees and other plants to provide shade, shelter, and other benefits to the plant. This can be done by using silvopasture systems, which involve the use of trees and other plants to provide shade, shelter, and other benefits to the plant. The physical environment can also be improved by using agroforestry, which involves the use of trees and other plants to provide shade, shelter, and other benefits to the plant. This can be done by using silvopasture systems, which involve the use of trees and other plants to provide shade, shelter, and other benefits to the plant.

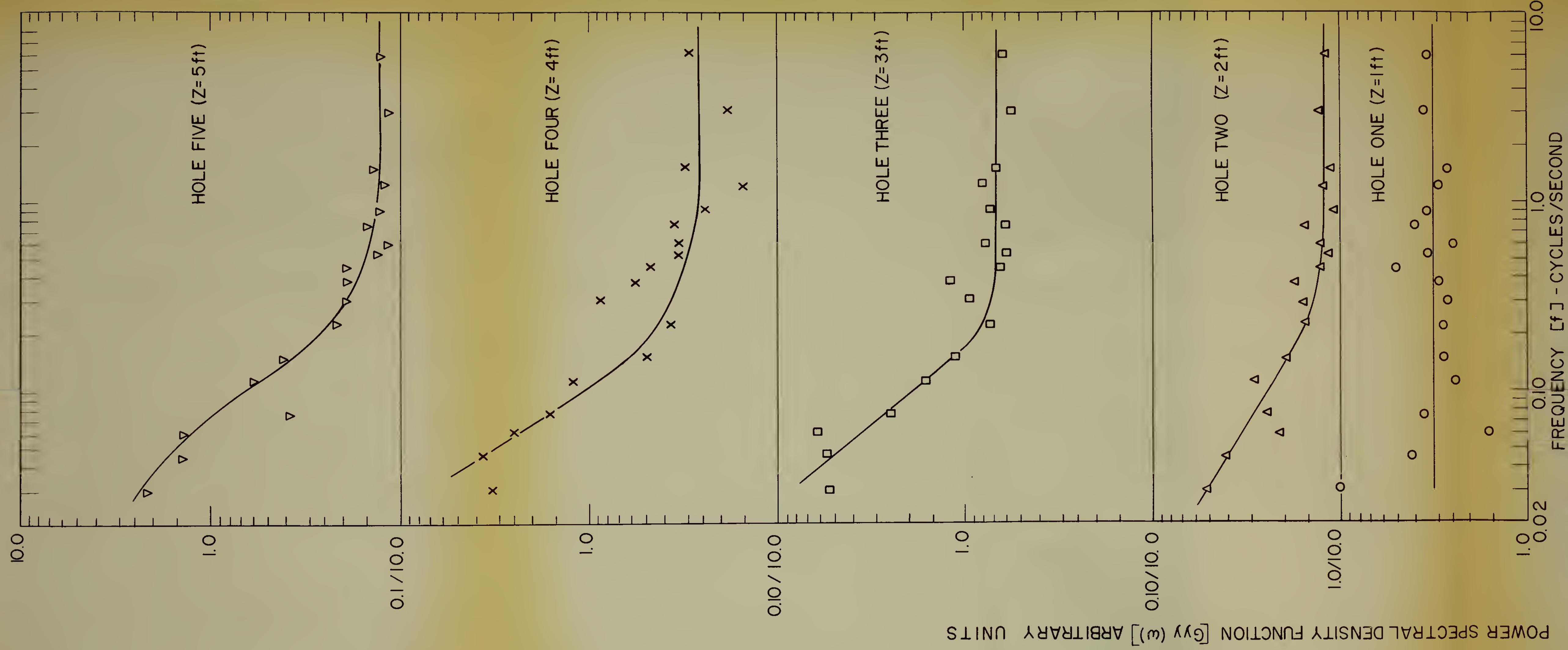


Figure 7. Comparison of power spectral density function with height in the subcritical assembly

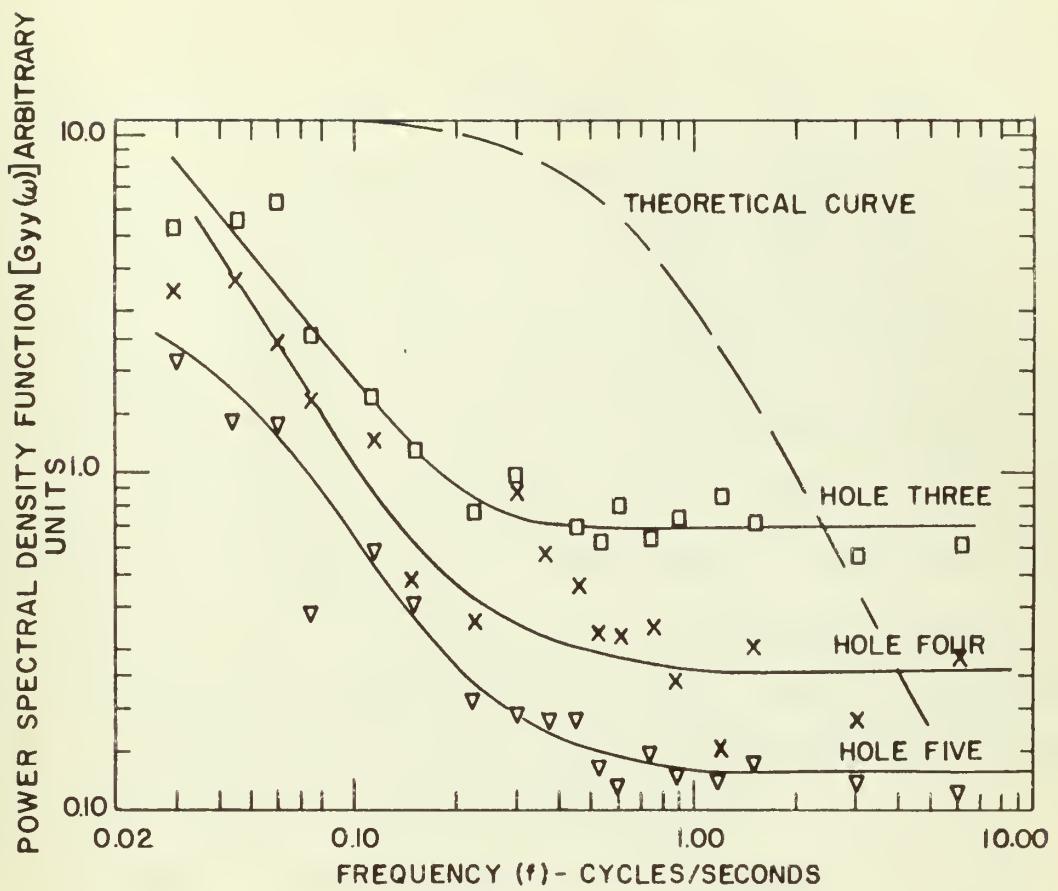


Figure 8. Comparison of measured with calculated power spectral density function

been attenuated by the transfer function of the assembly. Hole three ($z = 3$ ft) is 91.4 cm or approximately two diffusion lengths above the external source. The power spectral density function of the neutron flux at this level indicates that the transfer function of the assembly has operated on the flux to a much greater extent. Hole four ($z = 4$ ft) shows a maximum slope or attenuation of the power spectral density by the transfer function of the assembly. Hole five ($z = 5$ ft) shows a decreasing slope and a tendency to approach a break frequency toward the lower frequencies. This might be explained by a decrease in the " B " coefficient of Equation 57 which represents the level of assembly noise and is proportional to the square root of the power or flux level in the assembly. As " B " decreases, the instrumentation noise level approaches the assembly noise level, and the rounding off of the breaks in the curve tends to mask the true slope of the power spectral density function.

No definite break frequency was discernible from the data presented in Figure 7. The data for holes three, four, and five were replotted on Figure 8 along with a plot of the theoretical squared modulus of the transfer function from Equation 54. The theoretical equation was plotted using representative values of $\lambda = 0.08$ sec and $k = 0.5$. Comparison with the theoretical curves shows the slope of the power

spectral density of hole four to be approximately the same as that for the theoretical curve.

CONCLUSIONS

The results were inconclusive because the neutron break frequency was not determinable and as a result it was not possible to evaluate the ratio $\frac{1-k}{\lambda}$ exactly.

The best agreement between theory and experimental results was obtained at $z = 4$ ft = 122 cm. At this position, the slope of the attenuated power spectral density function was approximately the same as that predicted by theory. The value of the ratio $\frac{1-k}{\lambda}$ was determined to be approximately $0.4/0.08 = 5.0$. The results indicate that this ratio changes as z increases in value to beyond two diffusion lengths where it remains fairly constant until masked by the white noise of the detection chamber.

The evaluation of the transfer function of the sub-critical assembly by analysis of the power spectral density shows promise and should be pursued further.

Most notable was the increase in the number of individuals from 2000 to 2001. The 2001 figure is the best evidence that the new legislation has had a significant effect on the number of individuals. The number of individuals has increased steadily since 2000 and will continue to do so. The 2001 figure is also the best evidence that the new legislation has had a significant effect on the number of individuals. The 2001 figure is also the best evidence that the new legislation has had a significant effect on the number of individuals.

SUGGESTIONS FOR FURTHER STUDY

The plutonium-beryllium neutron sources utilized in this investigation were mounted on a pedestal beneath the assembly and were unmoderated except for the moderation by the graphite of the assembly. Interesting results might be obtained by encasing the sources in a block of paraffin moderator, and surrounding the moderated sources with graphite instead of the water shield. More significant results might be obtained by the use of a mechanical or electronic device for squaring and averaging the data. This means of data processing would allow much more rapid and accurate determination of the power spectral density, hence broadening the scope of investigation possible.

WIRTSCHAFTSWEISHEITEN

richt sich durch seine wirtschaftliche Machbarkeit nach Voraussetzung jenseits der Grenzen des technischen Fortschritts auf die Anwendung von Wissenschaft und Technik und auf die sozialen und ökologischen Folgen, welche diese Anwendung mit sich bringt. Wissenschaft und Technik müssen gleichzeitig nicht nur die gesetzliche Verpflichtung erfüllen, die sich durch die Anwendung nicht schädigen zu dürfen, sondern sie müssen auch die gesetzliche Verpflichtung erfüllen, die Anwendung nicht zu schädigen. Die Anwendung von Wissenschaft und Technik muss daher nicht nur die gesetzliche Verpflichtung erfüllen, die sich durch die Anwendung nicht schädigen zu dürfen, sondern sie muss auch die gesetzliche Verpflichtung erfüllen, die Anwendung nicht zu schädigen. Die Anwendung von Wissenschaft und Technik muss daher nicht nur die gesetzliche Verpflichtung erfüllen, die sich durch die Anwendung nicht schädigen zu dürfen, sondern sie muss auch die gesetzliche Verpflichtung erfüllen, die Anwendung nicht zu schädigen.

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103. The president of the U.S. has _____ 45
104. The president of the U.S. has _____ 45
105. The president of the U.S. has _____ 45

to know if we can make the same kind of statement about the other two groups.

an interesting model. It will exhibit both a 2D spatial 1700-arcsec \times 1700-arcsec low-resolution component and a 100-arcsec

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theoretical framework for measurement, which reflects well the empirical findings of different studies and summarizes the main research findings of the literature.

Overall, the theoretical framework is composed of three main components: the first component is the conceptual model of the relationship between the variables, the second component is the operationalization of the variables, and the third component is the statistical analysis.

The conceptual model of the relationship between the variables is based on the previous research findings, which suggest that there is a positive correlation between the variables. The operationalization of the variables is based on the previous research findings, which suggest that the variables can be measured using a Likert scale.

The statistical analysis is based on the previous research findings, which suggest that the variables can be measured using a Likert scale.

The results of the statistical analysis show that there is a positive correlation between the variables, which suggests that the variables can be measured using a Likert scale.

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APPENDIX

Sample calculations showing method of determining power spectral density.

Hole three ($z = 3$ ft)

$$\Delta f = 0.4 \text{ cps}, \bar{f} = 0.6 \text{ cps}$$

<u>i</u>	<u>y_i</u>	<u>y_i</u> ²
1	0.70	0.4900
2	-0.27	0.0729
3	0.70	0.4900
4	0.23	0.0529
5	0.40	0.1600
6	0.44	0.1936
7	1.25	1.5625
...
...
94	1.00	1.0000
95	-0.40	0.1600
96	0.43	0.1849
97	0.58	0.3364
98	1.30	1.6900
99	0.35	0.1225
100	0.90	0.8100

$$\sum y_i = 56.89 \quad \sum y_i^2 = 63.8859$$

$$\bar{y} = \frac{56.89}{100} = 0.5689$$

$$\bar{y}^2 = 0.323647$$

$$\overline{y^2} = \frac{63.8859}{100} = 0.638859$$

$$\sigma^2 = \overline{(y_1 - \bar{y})^2} = \overline{y^2} - \bar{y}^2.$$

$$= 0.638859 - 0.323647$$

$$= 0.315212$$

$$G_{yy}(\omega) = \sigma^2 / 4f = \frac{0.315212}{0.4} \cong 0.788 \frac{\text{mean square amplitude}}{\text{unit frequency}}$$

thesR785

Random neutron flux variations in a subc



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